

ECE 536 – Spring 2022

Homework #1 – Solutions

Problem 1) 1.4 of Textbook by Prof. Chuang

The direct (Γ -valley) bandgap energies are plotted vs. lattice constant for $\text{In}_{1-x}\text{Ga}_x\text{As}$ and $\text{Al}_x\text{In}_{1-x}\text{As}$ in Fig. 1.1. The intersections with the dashed vertical line represent the lattice-matched conditions to InP (lattice constant= 5.8688 Å). Note: At lattice match, the bandgap of AlInAs is larger than that of InP while the bandgap of GaInAs is smaller than that of InP.

It is worth noting that only AlInAs actually has an indirect X point with lower band gap energy than its direct Γ point shown. Code for generating plot shown is attached.

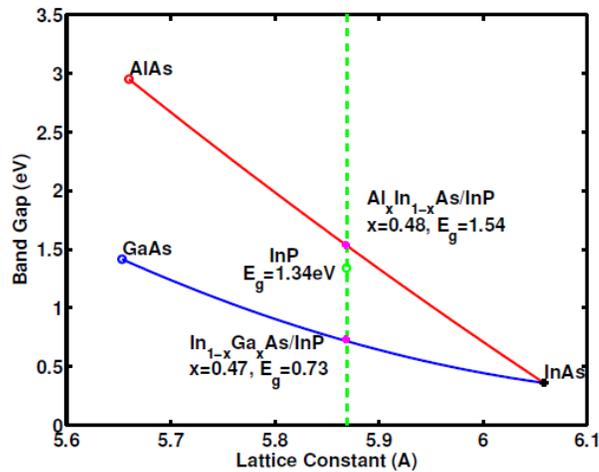


Figure 1.1: Plot of Band Gap vs. Lattice Constant showing InGaAs and AlInAs lattice match conditions with an InP substrate.

Problem 2)

To find the transition wavelength of these materials, we simply use the relation $\lambda (\mu\text{m}) = E_g(\text{eV})/1.24$, yielding the results shown in the table below.

	Si	GaAs	GaN	AlN	InP	GaP
E_g	1.124	1.424	3.44	6.2	1.344	2.272
λ	1.103	0.871	0.36	0.2	0.923	0.546

Using the same relationship as above, we find that the band gap energies for the given transition wavelengths to be

$$\lambda = 850 \text{ nm} \rightarrow 1.46 \text{ eV}$$

$$\lambda = 1.55 \mu\text{m} \rightarrow 0.80 \text{ eV}$$

To find the wavenumbers, we can just relate this to the wavelength

$$\text{wavenumber} = \frac{2\pi}{\lambda} = \frac{2\pi}{850 \text{ nm}} = 7.39 \cdot 10^4 \text{ cm}^{-1}$$

$$\text{wavenumber} = \frac{2\pi}{\lambda} = \frac{2\pi}{1.55 \mu\text{m}} = 4.05 \cdot 10^4 \text{ cm}^{-1}$$

Problem 3)

First, we find the mole fractions in $\text{In}_{1-x}\text{Ga}_x\text{As}$ and $\text{Al}_x\text{In}_{1-x}\text{As}$ that result in a material lattice matched to InAs. Using Appendices C1 and C2 of the text combined with Vegard's Law, we can write

$$\begin{aligned} a(\text{InP}) &= x \cdot a(\text{GaAs}) + (1-x) \cdot a(\text{InAs}) \rightarrow \text{In}_{0.53}\text{Ga}_{0.47}\text{As} \\ a(\text{InP}) &= x \cdot a(\text{AlAs}) + (1-x) \cdot a(\text{InAs}) \rightarrow \text{Al}_{0.48}\text{In}_{0.52}\text{As} \end{aligned} \quad (3.1)$$

From the band gap relations in Appendix C3, this results in

$$\begin{aligned} E_g(\text{In}_{1-x}\text{Ga}_x\text{As}) &= 0.36 + 0.505x + 0.555x^2 \rightarrow E_g = 0.718 \text{ eV} \\ E_g(\text{Al}_x\text{In}_{1-x}\text{As}) &= 0.36 + 2.35x + 0.24x^2 \rightarrow E_g = 1.533 \text{ eV} \end{aligned} \quad (3.2)$$

Obviously, then, the larger band gap material (AlInAs) is the barrier and the smaller band gap material (InGaAs) is the well, as shown in Fig. 3.1. To find the band edge discontinuities, use the information supplied in the initial problem, so that

$$\begin{aligned} \Delta E_c &= 0.70 \cdot \Delta E_g = 0.70 \cdot 815 \text{ meV} = 570.5 \text{ meV} \\ \Delta E_v &= 0.30 \cdot \Delta E_g = 0.30 \cdot 815 \text{ meV} = 244.5 \text{ meV} \end{aligned} \quad (3.3)$$

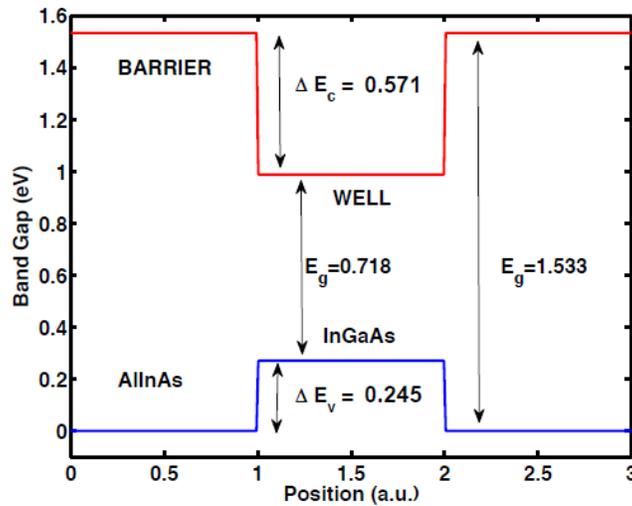


Figure 3.1: InGaAs/AlInAs quantum well structure.

Problem 4)

(A) ROUND TRIP Starting from Eqn. 1.2.1 in the text and taking the real part, we get

$$\begin{aligned} |r_1||r_2|e^{(G-\alpha)L} &= 1 \\ e^{(G-\alpha)L} &= \frac{1}{|r_1||r_2|} \\ G &= \alpha + \frac{1}{L} \ln \frac{1}{|r_1||r_2|} \end{aligned} \quad (4.1)$$

But $|r_i|^2 = R_i$, so

$$G = \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \quad (4.2)$$

And doing the same with the imaginary part, we see

$$\begin{aligned} e^{i2kL} &= e^{i2m\pi} \\ 2kL &= 2m\pi \end{aligned} \quad (4.3)$$

(B) FREQUENCY SPACING Starting from the phase condition above, the derivation is as follows:

$$\begin{aligned} 2kL &= 2m\pi \\ 2\pi n \frac{v_m}{c} L &= m\pi \\ 2\pi n \frac{v_{m+1}}{c} L &= (m+1)\pi \\ \Delta v = v_{m+1} - v_m &= \frac{c}{2nL} \end{aligned} \quad (4.4)$$

(C) WAVELENGTH SPACING This derivation follows the same pattern as that in Prob. 2. Starting with the phase relationship and differentiating both sides with respect to λ , we get

$$-\frac{2\pi nL}{\lambda^2} \Delta\lambda + \frac{2\pi L}{\lambda} \frac{\Delta n}{\Delta\lambda} = \Delta m\pi = \pi \quad (4.5)$$

Furthermore, assuming no dispersion, $\Delta n/\Delta\lambda \rightarrow 0$, we arrive at an expression for the corresponding wavelength spacing

$$|\Delta\lambda| = \frac{\lambda^2}{2nL} \quad (4.6)$$

(D) EXPERIMENTAL ANALYSIS If we choose the spectrum at 8mA, the first peak is located at 1500.65nm and the second is 1501.44nm. Thus $\Delta\lambda = 0.79\text{nm}$. Also, the refractive index of InP at 1500nm is around 3.165. Then the cavity length can be estimated as $450.33\mu\text{m}$ based on the formula in part (c). Using a different set of two peaks, you will get a slightly different answer. The reason is that in

reality, the material is slightly dispersive ($dn/d\lambda \neq 0$) and hence the mode spacing varies with the wavelength. The cavity length in Ref [47] is $398\mu\text{m}$ using Eq. 26 in that paper. The direct measurement gives $380\pm 20\mu\text{m}$. The comparison of the answer from part (c) and the experimental data shows the dispersion ($dn/d\lambda$) plays an important role in determining the cavity length. To obtain a better estimation, we should use the refractive index of InGaAsP (around 3.3) and take dispersion into account. Please refer to Eq. 7.6.21a.